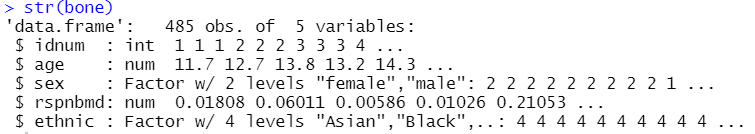
**Application of nonparametric regression on "Relative spinal bone mineral density measurements on 261 North American adolescents” data**

Data is taken from loon.data package on R software. From the definition :

From the web source: "Relative spinal bone mineral density measurements on 261 North American adolescents. Each value is the difference in spnbmd taken on two consecutive visits, divided by the average. The age is the average age over the two visits." The data are a repackaging and extension of the data of the same name from the now archived (in 2020) of the 2015 'ElemStatLearn' package of Kjetil B. Halvorsen.

For simplicity, writer will refer the data as “bone” data. This note will cover the results of applicating regression methods from parametric to nonparametric on the bone data. The methods which are used limited to linear regression (also included : polynomial regression) and smoothing regression (kernel and spline).

First of all, we’ll show the structure and some of the observations in the data.



Table

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As it can be seen from above, there are essentially 4 variables which consists of 2 categorical and 2 numerical variable. To emphasize again, this note will explore causal effect with the target variable is the “relative spinal bone mineral density” labeled rspnbmd with regression analysis. Also since this note will not touch the multivariate topic, the model would be built with age, sex, and ethnic as predictor variables for rspnbmd. Thus, the first model that is hypothesized is :

(model 1)

With is age, sex, and ethnic consecutively. The results from fitting the data to model 1 is as follow :

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Age is the only significant predictor. This would raise one question, are sex and ethnic really not a good predictor for rspnbmd? To answer this, ANOVA is used to model rspnbmd with these 2 categoric variables. The results :

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Only ethnic is significant and even after eliminating the interaction term, sex doesn’t seem to be a good predictor for rspnbmd on linear regression and ANOVA. Thus, for the categorical variable, writer concludes that Relative spinal bone mineral density is different in at least 2 ethnic groups in the bone data. Further, since in the linear regression only age is found to be significant predictor, the analysis will advance with only age as the predictor for rspnbmd.

Going back to regression model, since age is the only predictor left for rspnbmd, model 1 is reduced to

(model 2)

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Proceeding to analyze the result of model 2 : it can be seen that even though the model is significant, the value of R2 (which represent the variance of the data that can be explained by the regression) is pretty low. This implies that the regression ignores high amount of variance (thus ignores most of the information from the data). To see where the problem may come from, here is the plot between x = age and y = rspnbmd

Chart, scatter chart

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Subjectively speaking, the plot does seem to not follow a linear pattern. Also checking on homogeneity assumption :

Chart, scatter chart

Description automatically generated

Variance for the residual shows an increasing pattern as the fitted values increase. Thus a simple linear regression seems to underfit the bone data. While a lot of solution to this problem is available, writer focused on the plot between age and rspnbmd and decided to try fitting polynomial regression as the plot seems to show curve pattern.

After several trials, it is found that the optimal polynomial regression model for bone data is polynomial regression with degree of 4. To compare the fit for each polynomial regression model from degree 2 to 4, the following plots represent consecutively polynomial regression model degree 2, 3, and 4 applied to bone data :

Chart, scatter chart

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Chart, scatter chart

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Chart, scatter chart

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Curved line from the polynomial regression with degree of 4 seems reasonable enough to represent the relationship between age and relative spinal bone mineral density. As such, the current fitted model is :

(model 3)

And the summary of the model 3 fitted into bone data :

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Analyzing the summary, it is found that R2 after model improvement still doesn’t reach acceptable value for writer. Looking into the assumption, homogeneity is still the problem.

Chart, scatter chart

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Residual plot above shows similar pattern to that of residual plot from simple linear regression/model 2. Of course, same as before, there are several methods to overcome this problem. But, for the next part, this note is going to focus on the nonparametric solution.

Skipping details and theories, writer decide to fit a smoothed regression with 2 estimation methods : (1) kernel and (2) spline.

For the first part of nonparametric regression : kernel-smoothed regression. Kernel function to be used is gaussian/normal. Method on choosing the bandwidth used is based on cross-validation (CV). Writer uses 3 formula : maximum-likelihood, unbiased, and generalized. The report from each formula for the value of bandwidth is as follow :

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Generalized Cross-Validation

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After having bandwidth estimation based on CV, writer plot the smoothed kernel regression into the data plot.

Chart, scatter chart

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Chart, scatter chart

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Chart, scatter chart

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From the plot above, writer deducted that optimal bandwidth of kernel smoothing regression for bone data is around 0.9-1. Therefore, writer decided to choose bandwidth 0.95.

Chart, line chart

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